Arizona Mathematic Standards

Mathematics Curriculum Map

Fifth Grade

ARIZONA DEPARTMENT OF EDUCATION
HIGH ACADEMIC STANDARDS

Chandler Unified School District Standards

Fifth Grade – At a Glance

Curriculum Map

***Use Quarter 4 standards as opening routines in Quarters 1-3 to ensure exposure to all standards prior to AZMerit.**

Quarter 1	Quarter 2	Quarter 3	Quarter 4
Number & Operations in Base Ten **Operations & Algebraic Thinking**	**Number & Operations in Base Ten** **Measurement and Data**	**Number and Operations - Fractions**	**Measurement and Data** **Geometry** **Operations & Algebraic Thinking**
5.NBT.A.1 5.NBT.A.2 5.NBT.A.3 5.NBT.A.4 (Go Math) 5.NBT.B.6 5.NBT.B.7 (Go Math) 5.NF.B.3 (Go Math) 5.OA.A.1 (Go Math) 5.OA.A.2 (Go Math) 5.OA.B.4 5.NBT.B.5 (Fluency Standard)	5.NBT.A.2 5.NBT.A.4 (My Math) 5.NBT.B.6 (My Math) 5.NBT.B.7 5.MD.C.3 5.MD.C.4 5.MD.C.5 5.G.B.3 (Go Math) 5.G.B.4 (Go Math) 5.NBT.B.5 (Fluency Standard)	5.NF.A.1 5.NF.A.2 5.NF.B.3 (My Math) 5.NF.B.4 5.NF.B.5 5.NF.B.6 5.NF.B.7 5.MD.C.3 (Go Math) 5.MD.C.4 (Go Math) 5.G.B.3 (Go Math) 5.G.B.4 (Go Math) 5.NBT.B.5 (Fluency Standard)	5.MD.A.1 5.MD.B.2 5.G.A.1 5.G.A.2 5.G.B.3 (My Math) 5.G.B.4 (My Math) 5.NF.B.7 (Go Math) 5.OA.A.1 (My Math) 5.OA.A.2 (My Math) 5.OA.B.3 5.NBT.B.5 (Fluency Standard)

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

All quarters will include the Mathematical Practices

NOTE: Mathematical practices are interwoven and should be addressed throughout the year in as many different units and tasks as possible in order to stress the natural connections that exist among mathematical topics.

Grade 5 Key: NBT = Number and Operations in Base Ten, OA = Operations and Algebraic Thinking, NF = Number and Operations – Fraction, MD = Measurement and Data, G = Geometry

Fifth Grade Overview

Operations and Algebraic Thinking (OA)

- Write and interpret numerical expressions.
- Analyze patterns and relationships.

Number and Operations in Base Ten (NBT)

- Understand the place value system.
- Perform operations with multi-digit whole numbers and with decimals to hundredths.

Number and Operations—Fractions (NF)

- Use equivalent fractions to add and subtract fractions.
- Use previous understandings of multiplication and division to multiply and divide fractions.

Measurement and Data (MD)

- Convert like measurement units within a given measurement system.
- Represent and interpret data.
- Geometric measurement: Understand concepts of volume and relate volume to multiplication and to addition.

Geometry (G)

- Graph points on the coordinate plane to solve mathematical problems as well as problems in real-world context.
- Classify two-dimensional figures into categories based on their properties.

Standards for Mathematical Practices (MP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Fifth Grade Content Emphasis

Fifth Grade Content Emphasis	
Operations and Algebraic Thinking (OA)	
▲	Write and interpret numerical expressions.
▲	Analyze patterns and relationships.
Number and Operations in Base Ten (NBT)	
●	Understand the place value system.
●	Perform operations with multi-digit whole numbers and with decimals to hundredths.
Number and Operations – Fractions (NF)	
●	Use equivalent fractions to add and subtract fractions.
●	Use previous understandings of multiplication and division to multiply and divide fractions.
Measurement and Data (MD)	
▲	Convert like measurement units within a given measurement system.
▲	Represent and interpret data.
●	Geometric measurement: Understand concepts of volume and relate volume to multiplication and addition.
Geometry (G)	
▲	Graph points on the coordinate plane to solve mathematical problems as well as problems in real-world context.
▲	Classify two-dimensional figures into categories based on their properties.
● -Major Content ▲ -Supporting Content	
Major content (●) from the content emphasis section should account for approximately 70% of instructional time.	

Quarter 1

Essential Question(s):

- **How can you use place value, multiplication, and expressions to represent and solve problems?**
- **How can you divide whole numbers?**
- **How can you add and subtract decimals?**

80% of instructional time should be based on core materials (My Math, Go Math, Saxon); 20% may be based on additional resources.

Topic	Arizona Mathematics Standards	Resources
Fluency Standard	*To be taught throughout the year:* 5.NBT.B.5 Fluently multiply multi-digit whole numbers using a standard algorithm.	
*Standard to Supplement throughout Chapter 1	5.OA.B.4 Understand primes have only two factors and decompose a number into prime factors.	
Place Value, Multiplication, and Expressions	5.NBT.A.1 Apply concepts of place value, multiplication, and division to understand that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. 5.NBT.A.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. 5.NBT.B.5 Fluently multiply multi-digit whole numbers using a standard algorithm. 5.NBT.B.6 Apply and extend understanding of division to find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors. 5.OA.A.1 Use parentheses and brackets in numerical expressions, and evaluate expressions with these symbols (Order of Operations). 5.OA.A.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them (e.g., express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize that 3 x (18,932 + 921) is three times as large as 18,932 + 921, without having to calculate the indicated sum or product).	Go Math Chapter 1 **Additional Resources:** EngageNY Module 1 Topic A Topic B Topic C Illustrative Mathematics 5.NBT.A.1 Tasks 5.NBT.A.2 Tasks 5.NBT.B.5 Tasks 5.NBT.B.6 Tasks 5.OA.A.1 Tasks 5.OA.A.2 Tasks Georgia Standards – Unit 1

Quarter 1

Essential Question(s):

- **How can you use place value, multiplication, and expressions to represent and solve problems?**
- **How can you divide whole numbers?**
- **How can you add and subtract decimals?**

Divide Whole Numbers	5.NBT.B.6 Apply and extend understanding of division to find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors. 5.NF.B.3 Interpret a fraction as the number that results from dividing the whole number numerator by the whole number denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people, each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*	Go Math Chapter 2 **Additional Resources:** EngageNY Module 2 Topic E Topic F Topic G Illustrative Mathematics 5.NBT.B.6 Tasks 5.NF.B.3 Tasks
Add and Subtract Decimals	5.NBT.A.1 Apply concepts of place value, multiplication, and division to understand that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. 5.NBT.A.3 Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form. b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. 5.NBT.A.4 Use place value understanding to round decimals to any place. 5.NBT.B.7 Add, subtract, multiply, and divide decimals to hundredths, connecting objects or drawings to strategies based on place value, properties of operations, and/or the relationship between operations. Relate the strategy to a written form.	Go Math Chapter 3 **Additional Resources:** EngageNY Module 1 Topic D Illustrative Mathematics 5.NBT.A.1 Tasks 5.NBT.A.3 Tasks 5.NBT.A.4 Tasks 5.NBT.B.7 Tasks Georgia Standards – Unit 2 Inside Mathematics 5th Grade NBT Tasks

Quarter 1

Essential Question(s):

- **How can you use place value, multiplication, and expressions to represent and solve problems?**
- **How can you divide whole numbers?**
- **How can you add and subtract decimals?**

Multiply Decimals	5.NBT.A.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. 5.NBT.B.7 Add, subtract, multiply, and divide decimals to hundredths, connecting objects or drawings to strategies based on place value, properties of operations, and/or the relationship between operations. Relate the strategy to a written form.	Go Math Chapter 4 Started **Additional Resources:** EngageNY Module 1 Topic E Illustrative Mathematics 5.NBT.A.2 Tasks 5.NBT.B.7 Tasks Georgia Standards – Unit 3 Inside Mathematics 5th Grade NBT Tasks

***The Standards for Mathematical Practices are to be embedded throughout every chapter.**

Quarter 2

Essential Question(s):

- **How can you solve decimal multiplication problems?**
- **How can you solve decimal division problems?**
- **How do unit cubes help you build solid figures and understand the volume of a rectangular prism?**

80% of instructional time should be based on core materials (My Math, Go Math, Saxon); 20% may be based on additional resources.

Topic	Arizona Mathematics Standards	Resources
Fluency Standard	*To be taught throughout the year:* 5.NBT.B.5 Fluently multiply multi-digit whole numbers using a standard algorithm.	
Multiply Decimals	5.NBT.A.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. 5.NBT.B.7 Add, subtract, multiply, and divide decimals to hundredths, connecting objects or drawings to strategies based on place value, properties of operations, and/or the relationship between operations. Relate the strategy to a written form.	Go Math Chapter 4 Continued **Additional Resources:** EngageNY Module 1 Topic E Illustrative Mathematics 5.NBT.A.2 Tasks 5.NBT.B.7 Tasks Georgia Standards – Unit 3 Inside Mathematics 5th Grade NBT Tasks
Divide Decimals	5.NBT.A.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. 5.NBT.B.7 Add, subtract, multiply, and divide decimals to hundredths, connecting objects or drawings to strategies based on place value, properties of operations, and/or the relationship between operations. Relate the strategy to a written form.	Go Math Chapter 5 **Additional Resources:** EngageNY Module 1 Topic F Illustrative Mathematics 5.NBT.A.2 Tasks 5.NBT.B.7 Tasks Georgia Standards – Unit 3 Inside Mathematics 5th Grade NBT Tasks

Quarter 2

Essential Question(s):

- **How can you solve decimal multiplication problems?**
- **How can you solve decimal division problems?**
- **How do unit cubes help you build solid figures and understand the volume of a rectangular prism?**

Geometry and Volume	5.MD.C.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. 5.MD.C.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. 5.MD.C.5 Relate volume to the operations of multiplication and addition and solve mathematical problems and problems in real-world contexts involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes (e.g., to represent the associative property of multiplication). b. Understand and use the formulas $V = l \times w \times h$ and $V = B \times h$, where in this case B is the area of the base ($B = l \times w$), for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths to solve mathematical problems and problems in real-world contexts. c. Understand volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms, applying this technique to solve mathematical problems and problems in real-world contexts. 5.G.B.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. 5.G.B.4 Classify two-dimensional figures in a hierarchy based on properties.	Go Math Chapter 11 Started **Additional Resources:** EngageNY Module 5 Topic A Topic B Illustrative Mathematics 5.MD.C.5 Tasks 5.G.B.3 Tasks 5.G.B.4 Tasks Georgia Standards - Unit 6 Inside Mathematics 5th Grade MD Tasks

***The Standards for Mathematical Practices are to be embedded throughout every chapter.**

Quarter 3

Essential Question(s):

- **How can you add and subtract fractions with unlike denominators?**
- **How do you multiply fractions?**
- **What strategies can you use to solve division problems involving fractions?**

80% of instructional time should be based on core materials (My Math, Go Math, Saxon); 20% may be based on additional resources.

Topic	Arizona Mathematics Standards	Resources
Fluency Standard	*To be taught throughout the year:* 5.NBT.B.5 Fluently multiply multi-digit whole numbers using a standard algorithm.	
Geometry and Volume	5.MD.C.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. 5.MD.C.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. 5.G.B.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. 5.G.B.4 Classify two-dimensional figures in a hierarchy based on properties.	Go Math Chapter 11 Continued **Additional Resources:** EngageNY Module 5 Topic A Topic B Topic D Illustrative Mathematics 5.MD.C.3 Tasks 5.MD.C.4 Tasks 5.G.B.3 Tasks 5.G.B.4 Tasks Georgia Standards – Unit 5 Inside Mathematics 5th Grade G Tasks

Quarter 3

Essential Question(s):

- **How can you add and subtract fractions with unlike denominators?**
- **How do you multiply fractions?**
- **What strategies can you use to solve division problems involving fractions?**

Add and Subtract Fractions with Unlike Denominators	5.NF.A.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators (e.g., 2/3 + 5/4 = 8/12 + 15/12 = 23/12). 5.NF.A.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by using a variety of representations, equations, and visual models to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers (e.g. recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2).	Go Math Chapter 6 **Additional Resources:** EngageNY Module 3 Topic A Topic B Topic C Illustrative Mathematics 5.NF.A.1 Tasks 5.NF.A.2 Tasks Georgia Standards – Unit 4 Inside Mathematics 5th Grade NF Tasks
Multiply Fractions	5.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number and a fraction by a fraction. a. Interpret the product (*a/b*) x *q* as *a* parts of a partition of *q* into *b* equal parts. *For example, use a visual fraction model to show (2/3) x 4 = 8/3, and create a story context for this equation.* b. Interpret the product of a fraction multiplied by a fraction (*a/b*) x (*c/d*). Use a visual fraction model and create a story context for this equation. *For example, use a visual fraction model to show (2/3) x (4/5) = 8/15, and create a story context for this equation.* In general, (*a/b*) x (*c/d*) = *ac/bd*. c. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	Go Math Chapter 7 **Additional Resources:** EngageNY Module 4 Topic E Topic F Illustrative Mathematics 5.NF.B.4 Tasks 5.NF.B.5 Tasks 5.NF.B.6 Tasks Georgia Standards – Unit 4

Quarter 3

Essential Question(s):

- **How can you add and subtract fractions with unlike denominators?**
- **How do you multiply fractions?**
- **What strategies can you use to solve division problems involving fractions?**

Multiply Fractions Continued	5.NF.B.5 Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number; explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $\frac{a}{b} = \frac{n \times a}{n \times b}$ to the effect of multiplying $\frac{a}{b}$ by 1. 5.NF.B.6 Solve problems in real-world contexts involving multiplication of fractions, including mixed numbers, by using a variety of representations including equations and models.	Inside Mathematics 5th Grade NF Tasks
Divide Fractions	5.NF.B.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. Use the relationship between multiplication and division to justify conclusions. b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient.* Use the relationship between multiplication and division to justify conclusions (e.g., 4 ÷ (1/5) = 20 because 20 x (1/5) = 4). c. Solve problems in real-world context involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, using a variety of representations.	Go Math Chapter 8 Started **Additional Resources:** EngageNY Module 4 Topic G Illustrative Mathematics 5.NF.B.7 Tasks Inside Mathematics 5th Grade NF Tasks

***The Standards for Mathematical Practices are to be embedded throughout every chapter.**

Quarter 4

Essential Question(s):

- **How can you use line plots, coordinate grids, and patterns to help you graph and interpret data?**
- **What strategies can you use to compare and convert measurements?**

80% of instructional time should be based on core materials (My Math, Go Math, Saxon); 20% may be based on additional resources.

Topic	Arizona Mathematics Standards	Resources
Fluency Standard	*To be taught throughout the year:* 5.NBT.B.5 Fluently multiply multi-digit whole numbers using a standard algorithm.	
Divide Fractions	5.NF.B.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. Use the relationship between multiplication and division to justify conclusions. b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient.* Use the relationship between multiplication and division to justify conclusions (e.g., 4 ÷ (1/5) = 20 because 20 x (1/5) = 4). c. Solve problems in real-world context involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, using a variety of representations.	Go Math Chapter 8 Continued **Additional Resources:** EngageNY Module 4 Topic G Illustrative Mathematics 5.NF.B.7 Tasks Inside Mathematics 5th Grade NF Tasks
Algebra: Patterns and Graphing	5.MD.B.2 Make a line plot to display a data set of measurements in fractions of a unit (1/8, 1/2, 3/4). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.* 5.OA.B.3 Generate two numerical patterns using two given rules (e.g., generate terms in the resulting sequences). Identify and explain the apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane (e.g., given the rule "add 3" and the starting number 0, and given the rule "add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence).	Go Math Chapter 9 **Additional Resources:** EngageNY Module 6 Topic A Topic B Illustrative Mathematics 5.MD.B.2 Tasks 5.OA.B.3 Tasks 5.G.A.1 Tasks 5.G.A.2 Tasks

Quarter 4

Essential Question(s):

- **How can you use line plots, coordinate grids, and patterns to help you graph and interpret data?**
- **What strategies can you use to compare and convert measurements?**

Algebra: Patterns and Graphing Continued	5.G.A.1 Understand and describe a coordinate system as perpendicular number lines, called axes, that intersect at the origin (0 , 0). Identify a given point in the first quadrant of the coordinate plane using an ordered pair of numbers, called coordinates. Understand that the first number (*x*) indicates the distance traveled on the horizontal axis, and the second number (*y*) indicates the distance traveled on the vertical axis. 5.G.A.2 Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	Georgia Standards – Unit 7 Inside Mathematics 5th Grade OA Tasks 5th Grade G Tasks
Convert Units of Measure	5.MD.A.1 Convert among different-sized standard measurement units within a given measurement system, and use these conversions in solving multi-step, real-world problems. 5.MD.B.2 Make a line plot to display a data set of measurements in fractions of a unit (1/8, 1/2, 3/4). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*	Go Math Chapter 10 **Additional Resources:** Illustrative Mathematics 5.MD.A.1 Tasks 5.MD.B.2 Tasks Georgia Standards - Unit 6 Inside Mathematics 5th Grade MD Tasks

***The Standards for Mathematical Practices are to be embedded throughout every chapter.**

The Mathematical Practices: Narratives and Questions

Mathematics Practices		Narratives	Related Questions
Overarching habits of mind of a productive math thinker	**4.MP.1 Make sense of problems and persevere in solving them**	Mathematically proficient students explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. While engaging in productive struggle to solve a problem, they continually ask themselves, “Does this make sense?" to monitor and evaluate their progress and change course if necessary. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. Mathematically proficient students check their solutions to problems using different methods, approaches, or representations. They also compare and understand different representations of problems and different solution pathways, both their own and those of others.	• How would you describe the problem in your own words? • How would you describe what you are trying to find? • What do you notice about...? • What information is given in the problem? • Describe the relationship between the quantities. • Describe what you have already tried. What might you change? • Talk me through the steps you’ve used to this point. • What steps in the process are you most confident about? • What are some other strategies you might try? • What are some other problems that are similar to this one? • How might you use one of your previous problems to help you begin? • How else might you organize...represent... show...?
	4.MP.6 Attend to precision	Mathematically proficient students clearly communicate to others using appropriate mathematical terminology, and craft explanations that convey their reasoning. When making mathematical arguments about a solution, strategy, or conjecture, they describe mathematical relationships and connect their words clearly to their representations. Mathematically proficient students understand meanings of symbols used in mathematics, calculate accurately and efficiently, label quantities appropriately, and record their work clearly and concisely.	• What mathematical terms apply in this situation? • How did you know your solution was reasonable? • Explain how you might show that your solution answers the problem. • What would be a more efficient strategy? • How are you showing the meaning of the quantities? • What symbols or mathematical notations are important in this problem? • What mathematical language...,definitions..., properties can you use to explain...? • How could you test your solution to see if it answers the problem?

Actions and dispositions from NCSM Summer Leadership Academy, Atlanta, GA • Draft, June 22, 2011)
Most questions from all Grades Common Core State Standards Flip Book

The Mathematical Practices: Narratives and Questions

Mathematics Practices		Narratives	Related Questions
Reasoning and Explaining	**4.MP.2 Reason abstractly and quantitatively**	Mathematically proficient students make sense of quantities and their relationships in problem situations. Students can contextualize and decontextualize problems involving quantitative relationships. They contextualize quantities, operations, and expressions by describing a corresponding situation. They decontextualize a situation by representing it symbolically. As they manipulate the symbols, they can pause as needed to access the meaning of the numbers, the units, and the operations that the symbols represent. Mathematically proficient students know and flexibly use different properties of operations, numbers, and geometric objects and when appropriate they interpret their solution in terms of the context.	• What do the numbers used in the problem represent? • What is the relationship of the quantities? • How is _____ related to _____? • What is the relationship between _____and _____? • What does_____mean to you? (e.g. symbol, quantity, diagram) • What properties might we use to find a solution? • How did you decide in this task that you needed to use...? • Could we have used another operation or property to solve this task? Why or why not?
	4.MP.3 Construct viable arguments and critique the reasoning of others	Mathematically proficient students construct mathematical arguments (explain the reasoning underlying a strategy, solution, or conjecture) using concrete, pictorial, or symbolic referents. Arguments may also rely on definitions, assumptions, previously established results, properties, or structures. Mathematically proficient students make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). Students critique others by affirming or questioning the reasoning of others. They can listen to or read the reasoning of others, decide whether it makes sense, ask questions to clarify or improve the reasoning, and validate or build on it. Mathematically proficient students can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.	• What mathematical evidence would support your solution? • How can we be sure that...? / How could you prove that...? • Will it still work if...? • What were you considering when...? • How did you decide to try that strategy? • How did you test whether your approach worked? • How did you decide what the problem was asking you to find? • Did you try a method that did not work? Why didn't it work? Could it work? • What is the same and what is different about...? • How could you demonstrate a counter-example?

Actions and dispositions from NCSM Summer Leadership Academy, Atlanta, GA • Draft, June 22, 2011)
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The Mathematical Practices: Narratives and Questions

Mathematics Practices		Narratives	Related Questions
Modeling and Using Tools	**4.MP.4 Model with mathematics**	Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. When given a problem in a contextual situation, they identify the mathematical elements of a situation and create a mathematical model that represents those mathematical elements and the relationships among them. Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	• What number model could you construct to represent the problem? • What are some ways to represent the quantities? • What is an equation or expression that matches the diagram, number line, chart, table, and your actions with the manipulatives? • Where did you see one of the quantities in the task in your equation or expression? What does each number in the equation mean? • How would it help to create a diagram, graph, table...? • What are some ways to visually represent...? • What formula might apply in this situation?
	4.MP.5 Use appropriate tools strategically	Mathematically proficient students consider available tools when solving a mathematical problem. They choose tools that are relevant and useful to the problem at hand. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Students deepen their understanding of mathematical concepts when using tools to visualize, explore, compare, communicate, make and test predictions, and understand the thinking of others.	• What mathematical tools can we use to visualize and represent the situation? • Which tool is more efficient? Why do you think so? • What information do you have? • What do you know that is not stated in the problem? • What approach are you considering trying first? • What estimate did you make for the solution? • In this situation would it be helpful to use...a graph..., number line..., ruler..., diagram..., calculator..., manipulative? • Why was it helpful to use...? • What can using a _____ show us that _____may not? • In what situations might it be more informative or helpful to use...?

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The Mathematical Practices: Narratives and Questions

Mathematics Practices		Narratives	Related Questions
Seeing structure and generalizing	**4.MP.7 Look for and make use of structure**	Mathematically proficient students use structure and patterns to assist in making connections among mathematical ideas or concepts when making sense of mathematics. Students recognize and apply general mathematical rules to complex situations. They are able to compose and decompose mathematical ideas and notations into familiar relationships. Mathematically proficient students manage their own progress, stepping back for an overview and shifting perspective when needed.	• What observations do you make about...? • What do you notice when...? • What parts of the problem might you eliminate..., simplify...? • What patterns do you find in...? • How do you know if something is a pattern? • What ideas that we have learned before were useful in solving this problem? • What are some other problems that are similar to this one? • How does this relate to...? • In what ways does this problem connect to other mathematical concepts?
	4.MP.8 Look for and express regularity in repeated reasoning	Mathematically proficient students look for and describe regularities as they solve multiple related problems. They formulate conjectures about what they notice and communicate observations with precision. While solving problems, students maintain oversight of the process and continually evaluate the reasonableness of their results. This informs and strengthens their understanding of the structure of mathematics which leads to fluency.	• Explain how this strategy works in other situations? • Is this always true, sometimes true or never true? • How would we prove that...? • What do you notice about...? • What is happening in this situation? • What would happen if...? • Is there a mathematical rule for...? • What predictions or generalizations can this pattern support? • What mathematical consistencies do you notice?

Actions and dispositions from NCSM Summer Leadership Academy, Atlanta, GA • Draft, June 22, 2011)
Most questions from all Grades Common Core State Standards Flip Book

Table 1. Common Addition and Subtraction Problem Types/Situations. [1]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 - ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[2]**
Put together/Take Apart[3]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (“How many fewer?”): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has three fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with “fewer”): Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).
[2] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children that the = sign does not always mean **makes** or **results in** but always does mean **is the same quantity as.**
[3] Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10.

Table 2. Common Multiplication and Division Situations. [7]

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	3 x 6 = ?	**3 x ? = 18, and 18 ÷ 3 = ?**	**? x 6 = 18, and 18 ÷ 6 = ?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurements example:* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example:* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example:* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays [4]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example:* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example:* A rectangle has the area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example:* A rectangle has the area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare (Grade 4 ONLY)	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example:* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example:* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example:* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	a x b = ?	a x ? = p, and p ÷ a = ?	? x b = p, and p ÷ b = ?

[7] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

[4] The language is the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.